

6.2 Explain – Radicals and Rational Exponents - Notes

Essential Question: How do you simplify expressions with rational exponents?

Main Ideas/ Questions	Notes/Examples
What You Will Learn	<ul style="list-style-type: none"> Evaluate and simplify expressions with rational exponents. Rewrite expressions in rational exponent form. Solve real-life problems involving rational exponents.
What is a rational exponent?	<p>Definition: An exponent that can be written as a..... ratio (fraction)</p> <ul style="list-style-type: none"> Rational Exponent Form: $a^{\frac{m}{n}}$ <p>Practice: Using the calculator, evaluate the expressions.</p> <p>1. $32^{\frac{2}{5}} = 4$ 2. $343^{\frac{2}{3}} = 49$ 3. $-729^{\frac{5}{6}} = -243$</p>

Using the Laws of Exponents to Simplify Rational Exponents

Practice: Simplify the expressions. Use only positive exponents in your answer. (use calculator to Add fractions)

Multiply: (Multiply coefficients & Add exponents)

4. $5^{-\frac{3}{2}} \cdot 5^{\frac{7}{2}} =$
 $5^{(-\frac{3}{2} + \frac{7}{2})}$
 $= 5^2 = \boxed{25}$

5. $x^{\frac{1}{2}} (x^{\frac{1}{2}}) =$
 $x^{(\frac{1}{2} + \frac{1}{2})}$
 $= x^1 = \boxed{x}$

6. $2y^{\frac{3}{4}} \cdot 5y^{\frac{1}{2}} =$
 $10y^{(\frac{3}{4} + \frac{1}{2})}$
 $= \boxed{10y^{\frac{5}{4}}}$

7. $(2a^2b^{\frac{1}{4}})(-4a^{\frac{2}{3}}b^{\frac{1}{2}}) =$
 $-8a^{(2 + \frac{2}{3})}b^{(\frac{1}{4} + \frac{1}{2})}$
 $= \boxed{-8a^{\frac{8}{3}}b^{\frac{3}{4}}}$

Divide: (Subtract exponents)

8. $\frac{12d^{\frac{1}{2}}}{4d^{\frac{3}{4}}} =$
 $3d^{(\frac{1}{2} - \frac{3}{4})}$
 $= 3d^{-\frac{1}{4}} = \boxed{\frac{3}{d^{\frac{1}{4}}}}$

9. $\frac{125^3}{125^{\frac{8}{3}}} =$
 $= 125^{(3 - \frac{8}{3})}$
 $= \boxed{125^{\frac{1}{3}}}$

Exponent to Exponent: (Multiply exponents)

10. $(c^5d^2)^3 =$
 $c^{15}d^{3 \cdot 2}$
 $= \boxed{c^{15}d^6}$

11. $(16f^{\frac{1}{2}}g^{\frac{2}{3}})^{\frac{1}{2}} =$
 $= 16^{\frac{1}{2}}f^{\frac{1}{4}}g^{\frac{1}{3}}$
 $= \boxed{4f^{\frac{1}{4}}g^{\frac{1}{3}}}$

12. $(m^3)^{-1}(x^{\frac{1}{3}})^{\frac{1}{4}} =$
 $= m^{-3}x^{\frac{1}{12}}$
 $= \boxed{\frac{x^{\frac{1}{12}}}{m^3}}$

13. $(\frac{z^2}{5})^3 =$
 $\frac{z^2}{5^3} = \boxed{\frac{z^2}{125}}$

14. Find the area of the figure.

$(4x)^{\frac{3}{2}}$

$A = l w$
 $= (4x)^{\frac{3}{2}} (12x^{\frac{3}{4}})^{\frac{3}{4}}$
 $= 4^{\frac{3}{2}} x^{\frac{3}{2}} 12x^{\frac{9}{16}}$
 $= 8x^{\frac{3}{2}} 12x^{\frac{9}{16}} = \boxed{96x^{\frac{9}{4}}}$

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Rewriting Expressions in Rational Exponent Form

Examples: $\sqrt[3]{27} = 27^{1/3}$ $\sqrt[2]{5^3} = 5^{3/2}$ $(\sqrt[4]{x})^3 = x^{3/4}$

index → ↓ "2" when a # is not there

Property:

$$a^{m/n} = \sqrt[n]{a^m}$$

"R" (root)

Practice: Rewrite the expression in rational exponent form.

15. $\sqrt{10} = 10^{1/2}$

16. $\sqrt[5]{34} = 34^{1/5}$

17. $\sqrt[3]{x^2} = x^{2/3}$

18. $(\sqrt[5]{4})^3 = 4^{3/5}$

19. $(\sqrt[5]{x})^3 = x^{3/5}$

20. $(\sqrt[4]{x})^2 = x^{2/4} = x^{1/2}$

Practice: Simplify the expressions. Use only positive exponents in your answer.

$$21. \frac{2w^{-2}\sqrt[3]{x^5}}{(2w)^2} = \frac{2x^{5/3}}{w^2 \cdot 2^2 w^2} = \frac{2x^{5/3}}{4w^4} = \frac{x^{5/3}}{2w^4}$$

$$22. \frac{\sqrt[3]{x^2(x^3y^{-3})}}{y^3} = \frac{x^{2/3} \cdot x^3}{y^3} = \frac{x^{(2/3+3)}}{y^3} = \frac{x^{11/3}}{y^3}$$

23. The radius r of a sphere is given by the equation $r = \left(\frac{A}{4\pi}\right)^{1/2}$ where A is the surface area of the sphere. The surface area of a sphere is 1493 square meters. Find the radius of the sphere to the nearest tenth of a meter. Use 3.14 for π .

$$r = \left(\frac{1493}{4(3.14)}\right)^{1/2} = (118.9)^{1/2} = 10.9 \text{ meters}$$

24. Write the expression in exponential form. $2\sqrt[3]{a^1} + 3\sqrt[3]{a^1}$

$$2a^{1/3} + 3a^{1/3} = 5a^{1/3}$$

6.2 Radicals and Rational Exponents

In Exercises 1 and 2, rewrite the expression in rational exponent form.

1. $\sqrt{7}$

2. $\sqrt[4]{13}$

In Exercises 3 and 4, rewrite the expression in radical form.

3. $14^{1/4}$

4. $117^{1/6}$

In Exercises 5 and 6, evaluate the expression.

5. $\sqrt[3]{-125}$

6. $\sqrt[4]{81}$

In Exercises 7 and 8, rewrite the expression in rational exponent form.

7. $(\sqrt[4]{14})^3$

8. $(\sqrt[3]{-40})^5$

In Exercises 9 and 10, rewrite the expression in radical form.

9. $10^{3/5}$

10. $(-3)^{6/5}$

In Exercises 11–13, evaluate the expression.

11. $81^{3/4}$

12. $25^{3/2}$

13. $(-27)^{2/3}$

14. The area of a square patio is 49^3 square inches. Find the length of one side of the patio.

In Exercises 15–28, simplify the expression. Write your answer using only positive exponents.

15. $3^{-2/3} \cdot 3^{7/3}$

16. $(-2x^3)(5x^{\frac{1}{2}})$

17. $m^{-7/6} \cdot m^{1/4}$

18. $a^{\frac{1}{3}}b^2(a^{\frac{2}{3}}b)$

19. $\frac{7^{\frac{3}{4}}}{7^{-\frac{5}{4}}}$

20. $20k^{\frac{5}{4}} \div 5k^{\frac{1}{2}}$

21. $(\frac{1}{9})^{2/3} \cdot (\frac{1}{9})^{5/6}$

22. $(x^{\frac{5}{6}})^6$

23. $(25x^2)^{\frac{1}{2}}$

24. $(ab)^{\frac{1}{3}}(b)^{\frac{1}{3}}$

25. $(p^{-3})^{\frac{2}{5}}$

26. $(x^{\frac{1}{2}})^4 \sqrt[3]{x^3}$

27. $\frac{(x^{\frac{1}{2}})^4}{\sqrt[4]{x^3}}$

28. $\frac{\sqrt[8]{x^{12}}}{\sqrt{x}}$

29. The radius of a sphere is given by the equation $r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$, where V is the volume of the sphere.

Find the radius, to the nearest centimeter, of a sphere that has a volume of 268 cubic centimeters. Use 3.14 for π .

30. A math club is having a bake sale. Find the area of the bake sale sign.

